without surface charge; δ_0, initial amplitude of deformation of jet surface; E, electrical field stress; ρ_{s0}, initial surface charge density; v_r, v_x, velocity components; x, r, longitudinal and radial coordinates; t, time; H, Sh, E, dimensionless constants; x_*, r_*, t_*, φ_*, scale factors for coordinates, time, and electrical field potential; P, pressure; ρ_{s1}, perturbation of the surface charge density; , perturbation of the electrical field potential; η, surface deformation; r, radial coordinate; k, wave number; ε_0, absolute dielectric permeability; E_r, radial component of electric field; c_1, c_2, c_3, constants; $\ell = \gamma/Oh + k^2$; I_1, modified Bessel function; L, length of unbroken part of jet; ρ_s, surface charge density; ρ_c, charge density; j, current density; ψ, stream function; ΔL, increase in length of unbroken part of jet; v_j, jet velocity; ρ, liquid density.

LITERATURE CITED

1. A. F. Ginevskii and A. S. Dmitriev, Sb. Nauch. Tr. Mosk. Énerg. Inst., No. 149, 5-24 (1987).
2. V. A. Grigor'ev and A. I. Motin, Sb. Nauch. Tr. Mosk. Énerg. Inst., No. 185, 3-23 (1988).
3. V. M. Entov and A. L. Yarin, Itogi Nauki i Tekhniki, Ser. Mekh. Zhidk. Gaza, 17, 112-197 (1984).
4. V. V. Blazhenkov, A. F. Ginevskii, V. F. Gunbin, and A. S. Dmitriev, Inzh.-fiz. Zh., 55, No. 3, 41-418 (1988).
5. A. I. Motin and S. P. Tsybenko, Sb. Nauch. Tr. Mosk. Énerg. Inst., No. 149, 71-76 (1987).
6. A. F. Ginevskii, Sb. Nauch. Tr. Mosk. Énerg. Inst., No. 185, 54-58 (1988).
7. D. A. Savill, J. Fluid Mech., 48, Pt. 4, 815-827 (1971).
8. I. Adamchevskii, The Electrical Conductivity of Liquid Dielectrics [in Russian], Leningrad (1959).

EFFECT OF AN ELECTRIC FIELD ON THE CAPILLARY BREAKUP OF ELECTROLYTE JETS

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The results are presented of a theoretical and experimental investigation of the forced capillary breakup of jets of electrolytes in an electric field. It is shown that in some cases there is a nonmonotonic dependence of the length of the unbroken part of the jet on the electrical field stress.

Recently there have been considerable advances in the investigation and utilization of streams of charged monodisperse particles. Such streams have found wide utilization in the new chemical technologies, in electrical droplet spraying, printing, and marking devices, in cryogenic systems, etc. An important condition of the operation of such systems is the controllability of the droplet streams by means of an external field, so that it is necessary to place an electrical charge on the particles. A very promising method of placing a charge on a droplet of a conducting liquid is the induction charging of droplets during the forced capillary breakup of jets (FCBJ) of the liquid.

The induction charging of particles during the FCBJ is a complex process and depends on many parameters [1]; there are considerable experimental difficulties in connection with its study. Hence, in spite of quite a large number of theoretical and experimental papers dealing with the FCBJ in the absence of a field, there has been clearly insufficient investigation of the effect of an electrical field on the FCBJ process. Among the theoretical investigations it is necessary to mention papers [2-4], in which by way of a model there is consideration of the problem of the effect of an electric field on the capillary instability of an infinite cylinder, which does not correspond to the real FCBJ process in a limited jet. There are still fewer experimental studies, and the investigation of the effect of an electric field on the FCBJ has been pursued to a qualitative level only in them. For these rea-

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sons, the authors have been impelled to undertake a more detailed investigation of the effects of an electric field on the characteristics of the PCBJ.

For the experimental investigation of the features of the capillary breakup of electrolyte jets in an electric field use has been made of an automated experimental apparatus [5].

The breakup of the jet into droplets occurred in the field of a flat condenser, which was arranged so that one of the electrodes served as a shield, while the other had a voltage applied to it. The effect of the electric field on the capillary breakup was investigated for various efflux velocities v at a constant amplitude A and excitation frequency f. A constant voltage in the range 0-7 kV was placed on the charged electrode. For each rate of efflux, readings were taken of the initial length of the unbroken part of the jet L_j and its change in length ΔL_j as functions of the value of U. The measurements of L_j and ΔL_j were carried out by special optical systems to an accuracy of ± 5 μm.

The shape of the droplets and their charge were also recorded. A knowledge of the droplet charge makes it possible to determine for the known value of U the electrical field stress E at the point where the jet breaks up into droplets. In order to measure the charge used was made of a Faraday cylinder, an ampere voltmeter, and an interface connecting the ammeter with a computer. The determination of the average charge was carried out as follows. Over a specified interval of time the cylinder was connected to the ampere voltmeter, and the mean current was measured. By means of the interface, the information on the mean current was recorded in the computer memory. Simultaneously with the measurement of the mean current the frequency of formation of the droplets was also determined. In order to reduce the error caused by the leakage of a part of the cylinder current to earth, the cylinder was carefully insulated from the earth. The resistance of the insulation amounted to $\sim 10^{15}$ Ω. The error in establishing the mean charge was governed by the errors of the ampere voltmeter and amounted to ~0.5%.

In studying the FCBJ several regimes of the variation of L_j were observed, depending on the voltage being applied, the pulsation frequency of the generator, and the velocity of efflux. At large efflux velocities when the wave number was $k \lesssim 0.6$, a monotonic extension of the jet was observed with increase of the electrical field; for some potential differences the jet extension increased so much that the process of droplet formation began to occur after the charging electrode. At small efflux velocities a small shortening of the jet was first observed as the electrical field was increased, after which there was again a considerable extension. In order to explain the effects observed in the experiments let us consider the following effects which influence the behavior of the jet. In the first place, the longitudinal electrical field accelerates the flow of the liquid, and for this reason can increase the length of the unbroken part of the jet. In the second place, the fact is well known [2, 4] that an electrical field increases the increment of capillary instability, i.e., the rate of growth of perturbations on the jet surface, which must lead to a decrease in the length of the unbroken part of the jet. The competition between these two effects leads to the numerous regimes of jet flow in the electrical field.

In order to describe the features of the capillary breakup of jets of electrolyte in an electric field let us consider a jet of an ideal liquid of diameter D_j flowing with a velocity v_j out of a nozzle opening. The coordinate origin coincides with the point of efflux, and the x axis is directed along the jet. The description of the axisymmetric deformations of the jet surface occurring during capillary breakup will be carried out in a polar coordinate system (x, r).

In the presence of an electrical field which is external relative to the jet, the velocity of the jet v(x) will increase along the x axis according to Bernouilli's equation

$$\frac{v^2(x)}{2}+\frac{P_e(x)}{\rho}=\frac{v_j^2}{2}. \tag{1}$$

Thus, in order to establish the characteristics of the capillary instability of a jet of a conducting fluid in an external electrical field it is necessary to consider the problem of the instability of an accelerating jet.

Two scales are introduced for the longitudinal coordinate x: one is connected with the wave length of the pulsation of the jet surface (the corresponding coordinate is denoted by x), while the second is defined in terms of the characteristic scale of the growth of the perturbation along the jet [the corresponding coordinate is ($\xi = \varepsilon x$)]. It will be assumed that in the base state the electrical field stress depends only on the coordinate ξ at the surface of the jet and obeys the following model relationship (which is a good approximation for the electrode configuration used in the experiments according to numerical calculations of the electrical field distributions):

$$E_r \approx A\frac{\xi}{r}, \tag{2}$$

where A is a constant which can be determined from comparison with the experiments according to the formula

$$A = \frac{Q}{\pi\varepsilon_0 L_j \varepsilon}\left(\frac{4f}{3D_j^2 v_j}\right)^{2/3}. \tag{3}$$

The velocity potential Φ and the electrical field potential φ are introduced for the perturbations connected with the deformations of the jet surface. For these potentials we then have the system of equations

$$\Delta\Phi = 0, \tag{4}$$

$$\Delta\varphi = 0. \tag{5}$$

Scales are also introduced for the coordinates $x_* = r_* = R_j = D_j/2$, $\varphi^* = A\varepsilon R_j$, the time $t_* = R_j/v_j$, and the velocity $v_* = (\sigma/\rho R_j)^{1/2}$. In terms of the dimensionless variables Eqs. (4) and (5) retain their form, but the boundary conditions must be rewritten in linear form as follows:

on the jet axis, r = 0,

$$\frac{\partial\Phi}{\partial r} = 0, \tag{6}$$

at the efflux point, x = 0,

$$\eta = \delta_0 \cos\omega t;\ \Phi = 0, \tag{7}$$

at a distance from the surface, $r \to \infty$,

$$\varphi = 0, \tag{8}$$

on the jet surface, $r = 1 + \eta(x, t)$,

$$\frac{\partial\eta}{\partial t} + v\frac{\partial\eta}{\partial x} + \varepsilon v\frac{\partial\eta}{\partial\xi} = \varepsilon\frac{\partial\Phi}{\partial r}, \tag{9}$$

$$\frac{\partial\Phi}{\partial t} + v\frac{\partial\Phi}{\partial x} + \varepsilon v\frac{\partial\Phi}{\partial\xi} = \varepsilon\left[\eta + \frac{\partial^2\eta}{\partial x^2} + \frac{E_c R_j}{\varphi^*}E_r\left(\frac{\partial\varphi}{\partial r} + \frac{R_j}{\varphi^*}\eta\frac{\partial E_r}{\partial r}\right)\right], \tag{10}$$

$$\varphi + \frac{R_j}{\varphi^*}E_r\eta = 0, \tag{11}$$

where $v = v(\xi)/v_x$, $v(\xi)$ is given by Eq. (1), and $E_c = \varepsilon_0\varphi^{*2}/(\rho R_j v_*)$. Using the boundary conditions (6) and (7), a solution of the system of equations (4) and (5) is sought in the form

$$\Phi = \Phi_k(\xi)\exp[ik(\xi)x - i\omega t]\frac{I_0(kr)}{I_0(k)}, \tag{12}$$

$$\varphi = \varphi_k(\xi)\exp[ik(\xi)x - i\omega t]\frac{k_0(kr)}{k_0(k)}. \tag{13}$$

Based on the relationships (7), (9)-(13) a system of equations is obtained which describes to a linear approximation the forced capillary breakup of a jet of an ideally conducting electrolyte liquid in an electrical field:

$$\frac{\partial\eta}{\partial\xi} = \frac{k}{v(\xi)}\frac{I_1(k)}{I_0(k)}\Phi(\xi), \tag{14}$$

$$\frac{\partial\Phi}{\partial\xi} = \frac{\eta}{v(\xi)}(1 - k^2) + \frac{E_c\xi^2}{v(\xi)}\left[\frac{kk_1(x)}{k_0(k)} - 1\right]\eta, \tag{15}$$

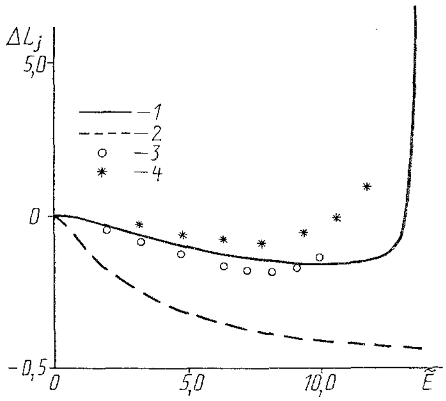

Fig. 1. The increase of the length of the unbroken part of the jet as a function of the value of the dimensionless electrical field: 1), 2): calculated results; 3), 4): experimental data; 1), 3): v = 4.24 m/sec; 2), 4): v = 3.44 m/sec. Dj = 200 μm, f = 5.23 kHz. The units for ΔLj are in mm.

$$k(\xi) = \omega/v(\xi) = \frac{\omega v_*}{\sqrt{v_j^2 - 2\varepsilon_0 E_r^2(\xi)/\rho}}. \qquad (16)$$

The initial conditions at $\xi = 0$ have the form

$$\eta = \delta_0, \ \Phi = 0. \qquad (17)$$

The system of equations (14)-(17) has been solved on a computer by the Runge–Kutta method. As a result of the calculations which were carried out, it has been shown that an increase in the electrical field stress E_r can lead to an extension or to a shortening of the unbroken part of the jet depending on the wave number (for small values of E_r). For large values of E_r the length of the unbroken part of the jet increased sharply.

By way of an example, Fig. 1 gives the calculated results and the experimental data for the dependence of ΔLj on the electrical field. From the curves which are given it can be seen that the model which has been proposed gives results which agree qualitatively with the experimental data. In the first place, the features of the FCBJ which are being considered occur at approximately the same scale values for the extension of the jet and for the value of the electrical field. In the second place, the dependence of the value of ΔLj on the numerous parameters (velocity, amplitude of the generator pulsation, electrical field stress, etc.) is in good agreement with the experimental data. The quantitative deviations which occur can be explained by the fact that the model which has been proposed does not take into account the nonlinear effects of the capillary breakup. In addition, the use of a model dependence of the electrical field stress on the coordinates does not always validly describe the experimental situation quantitatively, as shown by more detailed calculations. Apart from this, there exists a difficulty in principle in comparing the experimental data with the calculated results which consists of the impossibility of measuring the amplitude of the initial deformation of the surface δ_0. The tie-in to the experimental data was carried out by establishing δ_0 over the length of the unbroken part of the jet in the absence of a field. The dependence of δ_0 on the value of the applied voltage is not known.

NOTATION

P_e, electrical field pressure; ρ, liquid density; $\varepsilon = We^{-1/2}$, where We is the Weber number; Q, droplet charge; f, generator frequency; Lj, length of unbroken part of jet; ε_0, absolute dielectric permeability in a vacuum; $k(\xi) = \omega/v(\xi)$; I_0, k_0, Bessel functions of imaginary argument of the first and second types, respectively; v, jet velocity; ΔLj, increase in the length of the unbroken part of the jet in the electrical field; E, electrical field; k, wave number; Dj, jet diameter; x, longitudinal coordinate; vj, jet velocity at the nozzle cross-section; r, radial coordinate; ξ, slow longitudinal coordinate; Φ, velocity potential; φ, electrical field potential; Δ, Laplace operator; ω = 2πf; η, surface deformation; Rj, jet radius; E_r, radial component of electrical field; σ, surface tension; x_*, r_*, t_*, v_*, φ^*, scales for the coordinates, time, velocity, and electrical field potential; E_c, constant; Φ_k, φ_k, Fourier components of the velocity potential and electrical field potential; ω, angular frequency; δ_0, initial amplitude of surface deformation.

LITERATURE CITED

1. A. V. Bukharov, V. V. Blazhènkov, A. A. Vasil'ev, and S. N. Panasov, Inzh.-fiz. Zh., 60, No. 4, 646-651 (1991).
2. A. L. Huebner, J. Fluid Mech., 38, Pt. 4, 679-688 (1969).

3. C. I. Taylor, Applied Mechanics, Springer, New York (1969), pp. 382-388.
4. S. S. Nazin, A. N. Izotov, and V. B. Shikin, Dokl. Akad. Nauk SSSR, 283, No. 1, 121-125 (1985).
5. A. V. Bukharov and A. S. Sidorov, Sb. Nauch. Tr. MEI [Moscow Institute of Energetics], No. 185, 58-62 (1988).

FREEZING OF DROPS ON COOLED SURFACES

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The authors consider an approximate analytical solution of the problem of freezing of liquid drops on the cold surface of a semiinfinite body and a thin plate.

Freezing of liquid drops on a cooled substrate occurs in a number of contemporary and future technologies, e.g., cryodispersion technology, cryostorage of biological items in granules, and in the application of a controlled charged flux of solder drops to an electronic plate. In numerous applications it is important to know such characteristics of the freezing process as the crystallization time or the freezing rate and the temperature field in the drop during the freezing process.

The problem is formulated in these cases as follows. A drop in the form of a semiellipsoid of revolution is located on a cooled surface with temperature T at the initial time $\tau = 0$. This drop shape is the closest to the shape of granules obtained in existing technologies (Fig. 1). In most cases one can consider that the free part of the substrate and the curved surface of the drop are thermally insulated and all the heat from the drop is transferred to the substrate by heat conduction; there is no contact resistance between the drop and the substrate, and the initial temperature of the drop is the crystallization temperature (Fig. 2).

The temperature fields of the frozen part of the drop and in the substrate are described by the unsteady heat-conduction equations:

$$z >= 0, \quad \frac{\partial \Theta_d}{\partial \mathrm{Fo}_d} = \Delta \Theta_d; \tag{1}$$

$$z <= 0, \quad \frac{\partial \Theta_s}{\partial \mathrm{Fo}_s} = \Delta \Theta_s; \qquad \mathrm{Fo}_d = 0, \ \Theta_d = 1, \ \Theta_s = 0. \tag{2}$$

The temperature fields at the drop-substrate boundary are linked by the usual conjugate conditions:

$$\Theta_s = \Theta_d, \tag{3}$$

$$\left(\frac{\lambda_s}{\lambda_d}\right) \operatorname{grad} \Theta_s = \operatorname{grad} \Theta_d. \tag{4}$$

At the crystallization front in the drop we have the condition

$$\frac{\partial \Theta_d}{\partial n} = \frac{r \dfrac{\partial n}{\partial \mathrm{Fo}_d}}{[c_d(T_{cr} - T_\infty)]}. \tag{5}$$

It is scarcely possible to obtain an exact analytical solution of the Stefan problem of Eqs. (1)-(5) in the coupled form.

In [1] we proposed an approximate analytical method of solving this problem, based on introducing a curvilinear orthogonal coordinate system conforming to the shape of the object being analyzed.

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